Problem 1: The law of cosines states that $c^2 = a^2 + b^2 - 2ab \cos \theta$, where $a$, $b$, $c$ are the sides of a triangle and $\theta$ is the angle opposite the side of length $c$. Compute $\frac{\partial \theta}{\partial a}$.

\[
\begin{align*}
2ab \cos \theta &= a^2 + b^2 - c^2 \\
\frac{\partial}{\partial a} (2ab \cos \theta) &= \frac{\partial}{\partial a} (a^2 + b^2 - c^2) \\
2b \cos \theta - 2ab \sin \theta \frac{\partial \theta}{\partial a} &= 2a \\
\frac{\partial \theta}{\partial a} &= \frac{b \cos \theta - a}{ab \sin \theta - ab \sin \theta} \\
\frac{\partial \theta}{\partial a} &= \cot \theta \frac{a}{a} - \csc \theta \frac{b}{b}
\end{align*}
\]
Problem 2: Find the critical points of the function \( f(x, y) = 4x - 3x^3 - 2xy^2 \). Then, use the Second Derivative Test to determine whether they are local minima, local maxima, or saddle points (or state that the test fails).

\[
\begin{align*}
f_x &= 4 - 9x^2 - 2y^2 = 0 \\
f_y &= -4xy = 0
\end{align*}
\]

From the second equation, there are two cases. Case 1: \( x = 0 \). Then, \( 4 - 2y^2 = 0 \), so \((0, \sqrt{2})\) and \((0, -\sqrt{2})\) are critical points. Case 2: \( y = 0 \). Then, \( 4 - 9x^2 = 0 \), so \((\frac{2}{3}, 0)\) and \((-\frac{2}{3}, 0)\) are critical points.

\[
D = f_{xx}f_{yy} - (f_{xy})^2 \\
= (-18x)(-2x) - (-4y)^2 \\
= 36x^2 - 16y^2
\]

- \((0, \sqrt{2})\): \( D = 36x^2 - 16y^2 = -32 < 0 \). Saddle point.
- \((0, -\sqrt{2})\): \( D = 36x^2 - 16y^2 = -32 < 0 \). Saddle point.
- \((\frac{2}{3}, 0)\): \( D = 36x^2 - 16y^2 = 16 > 0 \). \( f_{xx} = -18x = -12 < 0 \). Relative maximum.
- \((-\frac{2}{3}, 0)\): \( D = 36x^2 - 16y^2 = 16 > 0 \). \( f_{xx} = -18x = 12 > 0 \). Relative minimum.